2019 Enrolment The 1st

Japan University Examination

Mathematics (Science)

Examination Date: November 2017

(90 min)

Do not open the examination booklet until the starting signal for the exam is given. Please read the following instructions carefully.
Please fill in the examinee no. and name below.

Instructions

1. The booklet contains 6 pages.
2. The answer sheet is one piece of one sided paper.
3. In the case that you notice there are parts in the booklet where the print is not clear or there are missing pages or misplaced pages, or the answer sheet is soiled, raise your hand to report to the invigilator.
4. There are 4 questions to be answered.
5. Fill the examinee no. and name in the answer sheet.
6. Use black pencil to write answers in the designated section in the answer sheet.
7. Memos and calculations can be written on the examination booklet.
8. When the signal to end the exam is given, check again to see that the examinee no. and name is filled in and submit the answer sheet and the examination booklet according to the invigilator's instructions.

Examinee'sNo.	Name

[1] Fill in the following blanks from [A] to [D'] with numbers.

(1) If $\alpha = 1 + 2i$, $\beta = 1 - 2i$ (i is an imaginary number), then

$$\alpha\beta = \boxed{\text{A}},\ \frac{5}{\alpha} + \frac{5}{\beta} = \boxed{\text{B}},\ \alpha^2 + \beta^2 = \boxed{\text{CD}}$$

(2) a, b are both positive constants, if straight line

$$y = ax + b, \qquad \cdots (*)$$

goes through (*) point (3, 1), then

$$b = \boxed{\text{EF}}\,a + \boxed{\text{G}}$$

If $-1 \leqq x \leqq 1$, and the minimum value of y is -7, then

$$a = \boxed{\text{H}},\ b = \boxed{\text{IJ}}$$

The solution of the inequation

$$-1 < ax + b < 1$$

is

$$\boxed{\text{K}} < x < \boxed{\text{L}}$$

(3) Consider that $1 \leq n \leq 6m + 4$ (n is an integer, m is a positive integer), and assume two sets A and B as

Set A : n is a multiple of 2

Set B : n is a multiple of 3

The number of n that belongs to set A is $\boxed{\text{M}}\,m + \boxed{\text{N}}$,

and the number of n that belongs to set B is $\boxed{\text{O}}\,m + \boxed{\text{P}}$

Then the number of n that belongs to the sum of set A and set B is

$$\boxed{\text{Q}}\,m + \boxed{\text{R}}.$$

（4） Consider that in triangle ABC，AB =3，AC =7，∠ABC =90°。

Set ∠BAC =θ,

then

$$\sin\theta = \frac{\boxed{S}\sqrt{\boxed{TU}}}{\boxed{V}}，\cos\theta = \frac{\boxed{W}}{\boxed{X}},$$

and

$$\sin（90° - \theta） = \frac{\boxed{Y}}{\boxed{Z}}.$$

Draw a line from point B that is perpendicular to line AC. When the intersection between the perpendicular line and line AC is H

then

$$BH = \frac{\boxed{A'}\sqrt{\boxed{B'C'}}}{\boxed{D'}}.$$

2 Fill in the following blanks from A to P' with numbers.

(1) Consider that k is a real number. Straight line

$$y = k(x-3)+4 \qquad \dots (*)$$

must go through point A (A , B) whatever the value of k is.

If straight line (*) intersects the y- axis at $y > 0$, then

$$k < \frac{C}{D}.$$

And, consider two points as point O(0, 0) and point B (−1, 3). If straight line (*) intersects line segment OB (except point O and point B), then

$$\frac{E}{F} < k < \frac{G}{H}$$

(2) Sequence $\{a_n\}$ 1, 5, 9, 13, ・・・

Sequence $\{b_n\}$ 2, 7, 12, 17, ・・・

Make a new sequence $\{c_n\}$ by arranging each item of the two sequences above as following:

$\{c_n\}$ 1, 2, 5, 7, 9, 12, 13, 17, ・・・

Express the value of the $2n$ item c_{2n} with n:

c_{2n} = I n − J

And

$$c_2 + c_4 + c_6 + \cdots + c_{200} = \text{KLMNO}$$

$$c_1 + c_2 + c_3 + \cdots + c_{200} = \text{PQRST}$$

(3) Consider that a is a positive real number. If the solution of equation

$$9^x - (a+1) \cdot 3^{x+1} + 8 = 0 \qquad \dots (*)$$

is $x = 0$, then

$$a = \text{U}$$

Set $t = 3^x$, then (*) can be expressed as

$$t^2 - \text{V}\, t + \text{W} = 0,$$

$$t = \text{X}, \ \text{Y} \ (\text{X} < \text{Y})$$

then, excluding $x = 0$, the solution of (*) is

$$x = \text{Z} \log_{\text{A'}} \text{B'}$$

(4) Consider that the equation

$$xy - 2x - 2y = 0 \qquad \ldots (*)$$

can be rewritten as

$$(x - \boxed{C'})(y - \boxed{D'}) = \boxed{E'}$$

The number of the sets of positive integers x, y satisfying (*) is $\boxed{F'}$.

And the number of the sets of positive integers x, y $(x < y)$ satisfying (*) is $\boxed{G'}$.

(5) Consider m is a positive integer, circle C and straight line l are both on xy plane:

Circle C: $x^2 + y^2 - 6x + 8 = 0$, Straight line l: $y = mx - 1$

The centre of circle C is point A, and the radius is $r(> 0)$, then

$$A(\boxed{H'}, \boxed{I'}), \ r = \boxed{J'}$$

Set the length of line segment drawn between point A and another point on straight line l is d, then minimum value of d is

$$d = \frac{|\boxed{K'}m - \boxed{L'}|}{\sqrt{m^2 + \boxed{M'}}}.$$

If circle C and straight line l intersect in 2 different points, then

$$\boxed{N'} < m < \frac{\boxed{O'}}{\boxed{P'}}$$

3 Fill in the following blanks from A to RS.

(1) Consider a is a constant, and the graph of quadratic equation $y = -x^2 + (4a + 4)x - 3a^2 - 10a - 3$ is C.

Then the apex of C is

$$(\boxed{\text{A}}\,a + 2,\ a^2 - \boxed{\text{B}}\,a + 1)$$

and it is a parabola.

When $a = -2$, parabola C intersects the x-axis at

$$x = \boxed{\text{CD}},\ \boxed{\text{E}}.$$

(2) Consider that if $3 \leqq x \leqq 4$, then the minimum value of quadratic function $y = -x^2 + (4a + 4)x - 3a^2 - 10a - 3$ is $m(a)$.

$$\text{If } a > \frac{\boxed{\text{F}}}{\boxed{\text{G}}}, \text{ then } m(a) = \boxed{\text{HI}}\,a^2 + \boxed{\text{J}}\,a.$$

$$\text{If } a \leqq \frac{\boxed{\text{F}}}{\boxed{\text{G}}}, m(a) = \boxed{\text{KL}}\,a^2 + \boxed{\text{M}}\,a - \boxed{\text{N}}.$$

$$\text{When } a = \frac{\boxed{\text{O}}}{\boxed{\text{P}}}, m(a) \text{ gives the maximum value } -\frac{\boxed{\text{Q}}}{\boxed{\text{RS}}}.$$

4 Fill in the following blanks from A to NO.

Consider that point P moves on plane xy according to the following rules.

Throw a dice for one time,

• for the x-axis : If the number that the dice showed is an even number, then plus 1, and if the number that the dice showed is an odd number, then minus 1.

• for the y-axis : if the number that the dice showed is 3,4 or 6,then plus 1,and if the number that the dice showed is 1,2 or 5, then minus 1.

For example, the coordination of point P is (4, 2), then throw the dice. If the dice shows 6,then point P will move to (A, B).

Set that point P was at (0,0) at first, after moving n times, the coordination of point P will be (X_n, Y_n).

(1)

The probability that $(X_1, Y_1) = (1, 1)$ is $\frac{C}{D}$

The probability that $(X_1, Y_1) = (-1, 1)$ is $\frac{E}{F}$.

The probability that $(X_1, Y_1) = (-1, -1)$ is $\frac{G}{H}$

The probability that $(X_1, Y_1) = (1, -1)$ is $\frac{I}{J}$

(2)

The probability that $(X_2, Y_2) = (2, 0)$ is $\frac{K}{L}$.

The probability that $(X_2, Y_2) = (0, 0)$ is $\frac{M}{NO}$